Assessing the Accuracy of Masses and Spatial Correlations of Galaxy Groups

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ABSTRACT

Two algorithms for the identification of galaxy groups from redshift surveys are tested by application to simulated data derived from N-body simulation. The accuracy of the membership assignments by these algorithms is studied in a companion to this paper (Frederic 1994). Here we evaluate the accuracy of group mass estimates and the group-group correlation function.

We find a strong bias to low values in the virial mass estimates of groups identified using the algorithm of Nolthenius & White (1987). The Huchra & Geller (1982) algorithm gives virial mass estimates which are correct on average. These two algorithms result in group catalogs with similar two-point correlations.

We find that groups in a CDM model have excessively large mass to light ratios even when the group richness distribution agrees with observations. We also find that our CDM groups are more strongly correlated than individual halos (galaxies), unlike the groups in the CfA redshift survey extension.

Subject headings: galaxies: clustering — galaxies: groups of

1. Introduction

Dynamical studies of groups of galaxies are important for probing the galaxy and the mass distributions in the universe over two orders of magnitude in mass, between the small scales probed by the internal velocities of individual galaxies (up to about $10^{12} M_{\odot}$) and the larger scales relevant to rich clusters of galaxies (over $10^{14} M_{\odot}$). Previous workers (see Paper I for references), many using different and often subjective criteria for defining groups, have identified groups, estimated their masses and other internal properties, and characterized their clustering properties. A general difficulty in this work has been the determination of reliable error estimates.

In this paper we are concerned with identifying galaxy groups and measuring their internal properties (masses, mass to light ratios, internal velocity dispersions, etc.) as well as their clustering properties. We study two algorithms for identifying galaxy groups from redshift surveys. The first was introduced by Huchra & Geller (1982, hereafter HG82) and used by the same authors in Geller & Huchra (1983, hereafter GH83) and again by Ramella, Geller & Huchra (1989,

hereafter RGH89). The second was proposed by Nolthenius & White (1987, hereafter NW87) as an improvement to the first. We test these algorithms by applying them to simulated redshift survey data obtained from an N-body simulation performed by Gelb (1992). A companion paper, Frederic (1994, hereafter Paper I), focuses on the accuracy of the membership assignments for each group finding algorithm. Here we study the accuracy of the group mass estimates and correlation function.

We optimize the group finding algorithms by applying them to the redshift survey catalogs constructed from our simulation. Once the limitations of the group finding algorithms have been determined, we can apply the optimized algorithms to real data. We use the first 6° declination slice of the Center for Astrophysics (CfA) redshift survey extension complete to $m_B = 15.5$ (Huchra, J.P., et al. 1990) The data were obtained electronically through the Astronomical Data Center of the National Space Science Data Center/World Data Center A for Rockets and Satellites at NASA Goddard Space Flight Center. We used the February 1992 version of the catalog, ADC catalog number 7144 (Huchra, J.P., et al. 1992).

Once a group catalog has been constructed and uncertainties in group membership have been quantified, group masses may be determined by application of the virial theorem or similar techniques (Heisler, Tremaine, & Bahcall, 1985). Application of these methods to simulated groups of known mass allows determination of the uncertainty inherent in the dynamical mass estimates.

Paper I contains descriptions of the N-body experiment and DENMAX, the method of galaxy identification in the simulation, as well as the method by which magnitude limited redshift catalogs were generated from the simulation. An important consideration dealt with in Paper I is the overmerging problem, common to high resolution N-body simulations of collisionless matter, in which small clumps of mass merge into extremely large clumps with almost no substructure. Specific group finding algorithms were then discussed and their accuracy studied. In this paper, section 2 describes the group finding algorithms. Section 3 is concerned with the internal properties of groups, such as masses and mass to light ratios. The group-group correlation functions are studied in section 4. Section 5 presents conclusions. An appendix containing definitions of various statistics follows.

2. Group Finding Algorithms

Groups are identified by both the HG82 algorithm and by the NW87 algorithm. Hereafter HG and NW will refer to the algorithms and to the groups constructed using the algorithms. Each operates on a list of galaxy angular positions and redshifts. Each pair of galaxies which are separated by less than some specified amount in both redshift and projected separation is considered linked. Each distinct set of mutually linked galaxies is a group.

The radial and transverse linking lengths for the HG algorithm are given by

$$D_L = D_0 \left[\int_{-\infty}^{M_V} \phi(M) dM \Big/ \int_{-\infty}^{M_{\rm lim}} \phi(M) dM \right]^{-1/3} , \tag{1}$$

$$V_L = V_0 \left[\int_{-\infty}^{M_V} \phi(M) dM \Big/ \int_{-\infty}^{M_{\rm lim}} \phi(M) dM \right]^{-1/3} , \tag{2}$$

where $M_V = m_{\rm lim} - 25 - 5\log(V/H_0)$ is the absolute magnitude of the brightest galaxy visible at a distance V/H_0 and V is the mean velocity of the pair. Similarly, $M_{\rm lim} = m_{\rm lim} - 25 - 5\log(V_F/H_0)$ is the absolute magnitude of the brightest visible galaxy at a fiducial distance V_F/H_0. D_0 and V_0 are the linking cutoffs at V_F, and $\phi(M)$ is the galaxy absolute magnitude (luminosity) function.

The same quantities for the NW algorithm are

$$D_L = D_0 \left[\frac{\int_{-\infty}^{M_V} \phi(M) dM}{\int_{-\infty}^{M_{\rm lim}} \phi(M) dM} \right]^{-1/2} \left[\frac{V_F}{V} \right]^{1/3} , \tag{3}$$

$$V_L = V_0 + 0.03(V - 5000 \text{ km s}^{-1}) . \tag{4}$$

In each algorithm, the linking lengths increase with redshift to account for the declining selection function. The primary difference between the two algorithms is in the scaling of the velocity linking length, which increases more slowly with redshift for the NW than for the HG algorithm. The algorithms are described in greater detail by the original authors and in Paper I.

These algorithms are applied to simulated redshift surveys constructed by two methods. In the first, which we call our "raw" catalog, each halo formed in the simulation is treated as a single galaxy. To construct our "breakup" catalog, halos which are too large as a result of overmerging are split into several galaxies based on an assumed mass to light ratio for clusters and the galaxy luminosity function. In either case, simulated galaxy luminosities are selected to fit the Schechter (1976) form of the luminosity function as determined by de Lapparent, Geller, & Huchra (1988) for the first slice of the CfA survey extension. The rank order of halo circular velocities is preserved, so that the nth brightest halo has the nth largest circular velocity. The details of these procedures are given in Paper I.

We construct group catalogs using three variants of each of the HG and NW algorithms, from ten (corresponding to ten different observers) simulated redshift surveys. Ten observers are used to produce enough groups to give good statistics. Of the three variants on the group finding algorithms, one is the basic algorithm, where groups are identified from an apparent magnitude limited redshift survey using only the observationally available coordinates of redshift, right ascension, and declination. We call the result a catalog of Vm groups, with V signifying the use of velocity (redshift) as the radial coordinate and m indicating that the source galaxy list was apparent magnitude limited. In order to isolate the effects of peculiar velocities on group identification we also construct our Rm catalog, in which real distance rather than velocity is

used as the radial coordinate. In this case we use the same link (D_L, eqs. [1] and [3]) in both the radial and transverse directions. Finally, we construct our RM catalogs, which are also based on true distances but have as their source an absolute magnitude limited galaxy list. The RM catalogs allow us to evaluate the effect of the magnitude limit on group properties. We construct RM catalogs using a fixed linking length. To summarize, for each of ten observers in two halo catalogs (breakup and raw), we construct three types (Vm, Rm and RM) of catalogs based on two grouping algorithms (HG and NW), for a total of 100 distinct group catalogs. (RM catalogs do not distinguish between HG and NW.) Unless explicitly noted, all galaxies and groups referred to are from simulation; real groups from the CfA survey are denoted as such.

3. Group Properties

Observers of galaxy groups have been particularly concerned with group mass to light ratios, under the assumption that the typical group mass to light ratio is representative of the universe as a whole. In particular, the median mass to light ratio of groups may be multiplied by the luminosity density of the universe to estimate the mean mass density. The luminosity function used to illuminate our simulated halos corresponds to a luminosity density of $1.035 \times 10^8\ h\ L_\odot\ \mathrm{Mpc}^{-3}$. Thus, a universal mass to light ratio of $1341h$ corresponds to $\Omega = 1$. We compute mass to light ratios for our simulated groups, but emphasize that any results based on mass to light ratios must be interpreted conservatively, as they are likely to be sensitive to the manner in which our dark halos were illuminated. For this reason we prefer to consider the accuracy of our group mass estimates, obtained with the standard virial theorem (Heisler et al. 1985), rather than mass to light ratios.

In order to determine the accuracy of our virial theorem mass estimates, we must compute the true masses of groups in the simulated catalogs. Our halo identification procedure, DENMAX, associates particles with peaks in the evolved density field. A further procedure removes those particles which are not gravitationally bound to a halo. The result is a mass for each N-body halo which we denote M_{halo}. Because the virial theorem is insensitive to mass which is not part of the system for much of its evolution, we expect the virial masses of the groups to measure the bound masses of the halos, plus any mass which may be bound to the group as a whole but not to any individual halo. In order to determine how much mass might be lurking in our simulated groups, unattached to halos, we counted the number of particles within one Abell radius ($1.5h^{-1}$ Mpc) of the centers of the illuminated halos in our raw halo catalog. We found that 69% of the mass in the simulation was within an Abell radius of a halo, and that 54% of the mass was bound to the halos. Therefore, at most 15% of the mass may be in groups but not bound to any halo. This estimate is certainly high, since it counts mass near all halos, not just halos in groups. And some fraction of this mass which is near but not bound to a halo is probably not bound to any group, despite being within an Abell radius, and therefore would not be reflected in the virial masses. So when

computing the true mass of a group, ignoring the mass which is not bound to any illuminated halo will probably result in an undercounting of the true mass by less than 10%.

In order to determine the total true mass of a simulated group we must know which of the galaxies fainter than the magnitude limit are to be included in the group. In the most general case, a given Vm or Rm group may contain some members which are common to a larger RM group, some other members which are equivalent to an entire RM group, and still other members which are not in any RM group. We choose to consider the true masses $M_{\rm true}$ of our Vm and Rm groups to be the sum of the masses of the RM groups which overlap them in whole or in part. That is, we sum the masses of the members of each RM group, which is complete down to an absolute magnitude $M_{B(0)} = -16.0$ (corresponding to a halo of 6 particles), and assign those masses to the Vm or Rm group which contains members of the RM group. We also computed, instead of the total mass of the RM components, the mass of the most massive RM component. In many cases these masses are identical. We prefer the total mass of the RM components to the largest mass component because the total mass includes those small RM groups that are below the magnitude limit in the Vm and Rm catalogs. Considering only the largest fraction of this mass would be ignoring mass which is near the Vm or Rm group and has contributed to the gravitational binding of the group.

3.1. Internal Properties of Simulated Groups

In Paper I we studied the accuracy of the group finding algorithms by comparing the memberships of redshift space groups to those of real space groups. It is possible, however, that accurate group memberships are not essential for computing certain statistics such as the mean or median of the distribution of a particular group property. RGH89 argue in favor of this point, claiming in particular that the median M/L of their most accurately identified groups was essentially the same as the median M/L for their entire sample of groups, including those contaminated by interlopers.

Here we consider the accuracy of the group catalogs, as opposed to the individual groups. To do so we compare the distributions of various group properties for our real space and our redshift space groups. Tables 1 through 4 present statistics of the simulated Vm group catalogs. They are the number of members N, the true group mass $M_{\rm true}$, the virial theorem mass estimate $M_{\rm VT}$, a measure of the accuracy of the mass estimate $M_{\rm VT}/M_{\rm true}$, the mean harmonic radius r_h, the dispersions in both velocity σ_v and true distance σ_r, the crossing time t_c, and the mass to light ratios $M_{\rm true}/L$ and $M_{\rm VT}/L$. The true distance dispersion σ_r and the crossing time t_c are presented as an indication of the likelihood of group accuracy. Small σ_r indicates compactness in space and small t_c means the groups have had time to virialize. Equations defining all these quantities are given in the appendix. The tables give the mean and the three quartiles of the distributions for

each quantity. Each entry in the tables gives a mean and a standard deviation obtained from our ten different observers. This same information is given for Rm groups in Tables 5 through 8, and for RM groups in Tables 9 and 10. Of course, not all of the above quantities are known for groups selected from a real redshift survey. Those that are are presented for the HG and NW groups in the CfA data in Tables 11 and 12.

The distributions of many of these statistics differ systematically between raw and breakup catalogs, between HG and NW grouping algorithms, and between Vm and Rm groups. When group identification is performed in redshift space in the apparent magnitude limited (Vm) galaxy catalogs, breakup groups tend to have higher true mass than do groups in the raw catalogs. Two effects are at work here. One is that some of the massive halos which were isolated and ungrouped in the raw catalog become rich, massive groups after breakup, thus raising the distribution of true group masses. The other is that due to the conservation of luminosity in the breakup procedure, faint, low mass groups in the raw catalogs disappear in the breakup case. These effects also cause the breakup groups to have higher median velocity dispersions and lower radii and crossing times than the raw groups. Group virial crossing times, proportional to r_h/σ_v, are therefore lower for the breakup groups. Since groups with crossing times greater than the Hubble time cannot be virialized, lower crossing times should generally result in more accurate virial masses. This explains why the virial mass estimates are better for the HG breakup groups than for the HG raw groups.

NW and HG groups differ predominantly in their velocity dispersions and therefore in their virial masses. Comparing raw HG to raw NW and breakup HG to breakup NW redshift space (Vm) groups, we see the primary difference between the two algorithms. The smaller velocity linking length used in the NW algorithm biases the group velocity dispersions, and hence the virial masses, to low values. The distributions of true masses for the HG and NW groups are almost identical, indicating that the two algorithms tend to pick out the same RM groups. However, by excluding the high velocity dispersion members of true groups, the NW algorithm leads to low virial mass estimates. As expected, this bias does not occur in the Rm catalogs. In fact, in real space the NW algorithm picks out groups with slightly higher velocity dispersions than the Rm HG groups.

After breakup, the virial masses of the HG groups are larger than before breakup, while the NW groups have smaller virial masses. The HG virial masses are in excellent agreement with the corresponding true masses. This is to be expected, since the massive halos which we broke up consisted originally of virialized clumps of particles. The NW algorithm, on the other hand, does a poor job estimating group masses because its velocity linking length is too small.

The Rm group masses, both true and virial, are not sensitive to the breakup procedure. Breakup Rm groups have smaller projected and harmonic radii and larger velocity dispersions than do raw groups, but these effect cancel each other out in the calculation of the virial mass.

Groups identified in an absolute magnitude limited halo sample using a fixed linking length

(RM) have significantly different properties than the groups found in the apparent magnitude limited halo catalogs (Vm and Rm). Statistics for the RM groups are given in Tables 9 and 10. These groups were identified in halo catalogs with an absolute magnitude limit $M_{B(0)} < -16$; this is equivalent to volume limiting out to 1000 km s^{-1}. The linking length used was $0.55h^{-1}$ Mpc, but the results are similar for a linking length of $0.27h^{-1}$ Mpc, which is equal to the HG transverse linking length D_L at 1000 km s^{-1}. The differences seen between the raw and breakup Vm and Rm catalogs are also present in the RM catalogs; e.g., the fact that breakup groups are more massive than raw groups. But the distribution of RM group masses is significantly lower than those of the Vm and RM masses, due to the fact that so many more faint halos exist in the RM catalog.

Unfortunately, some and perhaps all of the differences between the RM and Vm or Rm catalogs are due to the effect of the apparent magnitude limit and hence are sensitive to the method by which our simulated halos were illuminated. The reader may note that the virial mass to light ratios in raw RM groups are high compared to Vm or Rm groups; perhaps this means that we underestimate the true $M_{\rm VT}/L$ of groups in the real data. We cannot make this claim, however, because of the manner in which our halos were illuminated. The simulation produces many more faint halos than are observed if we assume a constant mass to light ratio for illuminating all halos. Instead, we forced our luminosity function to fit that of the CfA data. As a result, the luminosity function of halos rises more slowly than the mass function at the low end. Consequently, our low mass halos were assigned even lower luminosities, resulting in a high M/L for faint halos. This effect is illustrated in Figure 1, which shows $M_{\rm halo}/L$ as a function of $M_{\rm halo}$. The spread in the relation is because assigned luminosities are rank ordered by circular velocity, which correlates well but not exactly with $M_{\rm halo}$. The striped effect arises from the quantization of true halo masses in units of the particle mass. Since our low mass, faint halos have high mass to light ratios, our faint groups present in the RM catalogs will can be expected to have high mass to light ratios also.

3.2. Internal Properties of CfA Groups

Although it cannot help us judge the accuracy of the group finding algorithms, it is nonetheless interesting to look at the properties of the HG and NW groups identified in the CfA data. These data are given in Tables 11 and 12. Again we see that the NW groups have lower velocity dispersions and virial masses than the HG groups. The distributions of the three independent group properties r_h, σ_v and $M_{\rm VT}/L$ are all narrower (as measured by the difference between the third and first quartiles of the distribution), indicating that NW groups are a less diverse class of objects than are HG groups. Again, this comes at the expense of biased velocity dispersions. In addition to having smaller virial masses, NW groups in the CfA data tend to be smaller in spatial extent than HG groups.

NW groups in the CfA data we analyze have extremely low values of $M_{\rm VT}/L$. NW87 find a median mass to light ratio of $220h$ in the first CfA survey, whereas we find a value of only $110h$

in the CfA extension. The discrepancy here is probably due to the relative depths of the original CfA survey and its extension, which is one magnitude deeper. Almost all of the groups found by NW87 were less distant than 8000 km s^{-1}, which is about where the HG and NW scalings for the velocity linking parameter begin to diverge rapidly. As a result, the groups found by NW87 in their more shallow galaxy sample were not significantly affected by the biased velocity dispersions we see here.

4. Group-Group Correlation Functions

The correlation function $\xi(s)$ is another diagnostic by which we can compare the NW and HG algorithms, and at the same time study the effects of peculiar velocities on the group correlation function by comparing Vm groups to Rm and RM groups. We estimate the correlation function as

$$\xi(s) = \frac{n_R}{n_D}\frac{N_{DD}(s)}{N_{DR}(s)} - 1\,, \tag{5}$$

$$s = \frac{(V_i^2 + V_j^2 - 2V_iV_j\cos\theta_{ij})^{1/2}}{H_0}\,, \tag{6}$$

where V_i and V_j are the radial velocities of two galaxies with angular separation θ_{ij} and H_0 is Hubble's constant. In order to account for edge effects, a catalog of randomly distributed points with geometry and selection function identical to the real or simulated data must be generated. Then n_D and n_R are the number of points in the data and the random catalog, respectively, $N_{DD}(s)$ is the number of pairs in the data separated by redshift distance s, and $N_{DR}(s)$ is the number of pairs, one from the data and one from the random catalog, separated by s. We denote the galaxy-galaxy luminosity function as $\xi_{\rm gg}$ and the group-group function as $\xi_{\rm GG}$.

Calculating the correlation functions requires constructing catalogs of randomly distributed points in the same volume as the real or simulated data and with the same selection function. We use the same random catalogs for computing both ξ_{gg} and ξ_{GG}. A comparison of the redshift distributions of simulated groups to that of simulated galaxies revealed good agreement, justifying our implicit assumption of the similarity of the simulated galaxy and group selection functions. In order to minimize the statistical noise from the random catalog, we computed $\xi(s)$ for ten separate simulated catalogs corresponding to our ten different observers, using a different random catalog for each observer. These results were then averaged.

We were also concerned with the effects the edges of the sample volume might have on the group distribution, since small groups straddling the edge of the volume may not be identified. In fact, the simulated group distribution does cut off abruptly at about 0.1° from the edges of the sample volume. We compensated for this in our random catalogs by masking out all points within 0.1° of the edges of the sample when computing $\xi_{\rm GG}$, and found this to have almost no effect on the resulting $\xi_{\rm GG}$ correlation functions.

Figure 2 shows $\xi(s)$ for halos and for groups identified in different ways from the raw catalogs. The corresponding figure for the breakup case is Figure 3. The general relationships between the different $\xi(s)$ curves for the breakup catalogs and the raw catalogs are quite similar. However, as the correlation function of the breakup halos is higher than for the raw case, the different $\xi(s)$ curves are squeezed together in Figure 3.

In both the apparent and absolute magnitude limited samples, the group-group correlation function is higher than that of the galaxies, as one would expect if clustering is hierarchical (Bahcall & Soneira 1983; Kaiser 1984; Bahcall 1988). Ramella et al. (1990) find this not to be the case in their analysis of three 6° slices of the CfA survey extension. Unfortunately, the single slice of that same data which we are studying does not contain enough groups from which to construct a reliable correlation function.

For the simulated raw and breakup catalogs, the correlation functions have similar slopes but differ in their amplitudes. First note the behavior of $\xi_{\rm gg}$, which is the main point of difference between the raw halos and the breakup halos. $\xi_{\rm gg}$ for the apparent magnitude limited (Vm) raw catalog lies between the $\xi_{\rm gg}$ curves computed from raw catalogs which were absolute magnitude limited at $M_{B(0)} \leq -16$ and $M_{B(0)} \leq -20$. $M_{B(0)} = -16$ is the faintest halo visible at 1000 km s^{-1}, while $M_{B(0)} = -20$ is the faintest halo visible at 6500 km s^{-1}, which is the median redshift for our groups. This behavior occurs because the correlation amplitude of our simulated halos increases with luminosity. This property has been found in the original CfA survey (Hamilton 1988). As one looks to more distant galaxies (or halos), one sees brighter objects with intrinsically stronger clustering. The correlation function measured for an apparent magnitude limited sample like our Vm catalog is a weighted average of the correlation functions of objects of different luminosities. This does not explain the fact that for the breakup groups, the Vm halos are not any more correlated than the RM halos which have been absolute magnitude limited at -16. The reason for this is that the correlation functions are squeezed together in the breakup case until their error bars overlap. In either case, the correlation amplitude of the halos in the apparent magnitude limited catalog is larger than that of the halos in the absolute magnitude limited catalog by about 0.1 dex, or 25%.

We find the same amplification of the correlation length in the Vm groups relative to the RM groups. Again, this is because only groups containing bright and relatively strongly clustered halos appear in much of the volume of the Vm and Rm samples. The amount of the increase in the correlation length appears to be about the same, or possible a little larger, for the halos than for the groups.

Because there are fewer NW than HG groups, the correlation functions for the NW groups are noisier than but otherwise indistinguishable from those of the HG groups. This indicates that the two algorithms are equally suited to determining the group-group correlation function. We noted above that the similarity of the distributions of true mass between the HG and NW groups also indicated that they tend to locate the same objects, although the NW algorithm results in

biased virial mass determinations.

Although the Rm groups appear to be more strongly correlated than the Vm groups, these curves are noisy and their error bars (not shown) do overlap. This behavior makes sense when one considers that the Vm groups typically contain an Rm group plus some interlopers. Then the spatial distribution of Vm groups is the same as that of the Rm groups with a few spurious groups added at uncorrelated positions. Therefore it appears that the use of velocities rather than true distances in identifying groups is adequate for the computation of a redshift space group-group correlation function.

5. Conclusions

Comparing the internal properties of groups identified by the HG and NW methods reveals a significant bias in the NW groups in deep surveys toward low velocity dispersions and virial masses. This bias was not significant in the original application of the NW algorithm (NW87) to the original CfA survey due to the relatively shallow depth of that survey. Nolthenius (1993) reports that in fact, a slight negative trend of M/L with distance was seen in NW87, as we would expect, and that the effect was more pronounced in his flow-model corrected group catalog. This effect is simply interpreted as the result of group truncation due to an overly restrictive velocity linking criterion, whereby masses of groups at large redshift are systematically underestimated. It is seen here in both our simulated catalogs and in the real data from the CfA survey extension. For the HG algorithm, on the other hand, individual mass estimates for the breakup groups are unbiased, with the median $\log_{10}(M_{\rm VT}/M_{\rm true}) = 0.003 \pm 0.075$. The virial masses of raw HG groups are moderately overestimated, with the median $M_{\rm VT}$ about 50% larger than the median $M_{\rm true}$, though the error bars on each of these median values is almost as large as the difference between them. Because our breakup galaxy catalogs match the clustering of real (CfA) galaxies better than do our raw catalogs, we expect the accuracy of virial masses for real groups to be close to that seen here for simulated breakup groups when the HG algorithm is applied.

Diaferio et al. (1993) find that the spread in group masses found by RGH89 is consistent with the spread in virial mass estimates of a single collapsing loose group viewed from different angles. We find that the distribution of virial masses is broader than that of true masses for both our raw and breakup HG groups. Projection effects may be responsible for this spread.

We find that the NW and the HG algorithms produce groups with similar correlation functions $\xi_{\rm GG}(s)$, although the HG algorithm is slightly better suited to this task because it produces more groups, for better statistics. Comparing $\xi_{\rm GG}$ computed from our Vm and Rm groups indicates that the use of velocities rather than true distances for identifying groups does not affect group clustering as measured by the correlation function $\xi_{GG}(s)$.

The virial masses of our simulated groups are higher for our breakup groups than for raw

groups, and in both cases are much higher than the virial masses of CfA groups. This may be an important shortcoming of the CDM model, as this is a purely dynamical effect, not one which can be ascribed to our halo illumination technique or to the lack of gas dynamics in the simulation. Our CDM groups are also more strongly correlated than individual CDM halos, contrary to the finding of Ramella et al. (1990) for groups found in the first two slices of the CfA redshift survey extension.

Properties of our RM groups differ significantly from those of our Vm and Rm groups. This fact should serve as a reminder to those performing cosmological structure simulations that it is important to compare simulated data to observations as accurately as possible, by mimicking the observation procedure and including important effects such as a flux limit.

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6. Appendix: Definitions of Group Properties

The quantities in Tables 1 through 12 are defined as follows: The number of members in a group is N. The true mass measured from the simulation is denoted $M_{\rm true}$ and is computed from the largest number of those particles grouped together by DENMAX which are mutually gravitationally bound. The group mass quoted is the sum of the masses of the halos in all RM groups whose membership totally or partially overlaps the membership of the Vm or Rm group.

A statistical correction is applied to group luminosities to account for group members which are below the magnitude limit of the catalog:

$$L = L_0 \frac{\int_0^\infty L\phi(L)dL}{\int_{L_V}^\infty L\phi(L)dL}\,, \tag{7}$$

where L_0 is the sum of the luminosities of the visible members and L_v is the minimum luminosity visible at a redshift V. When computing the mass to light ratio $M_{\rm true}/L$ we use true distances of individual group members to convert their apparent magnitudes to luminosities and the true distance of the group center to make the group luminosity correction. For the $M_{\rm VT}/L$ statistic we used redshift distances for computing both individual member luminosities and the group luminosity correction. We also tested using the group's mean redshift distance for computing individual members' luminosities from their magnitudes. This made no significant difference in the median group mass to light ratio.

The remaining quantities are defined as by NW87. The mean harmonic radius, r_h, is given by

$$r_h = \frac{\pi}{2}\frac{V}{H_0}N(N-1)\Big[\sum_{i<j}\frac{1}{\tan\theta_{ij}/2}\Big]^{-1} . \tag{8}$$

The dispersions in both velocity and true distance, σ_v and σ_r, are

$$\sigma_s = \Big[\frac{1}{N-1}\sum_{i=1}^{N}(s_i - \langle s\rangle)^2\Big]^{1/2} , \tag{9}$$

where s can refer to either redshift (v) or true (r) distance and $\langle s\rangle$ is the arithmetic mean for the group. The virial crossing time, in units of the Hubble time, is

$$H_0 t_c = \frac{2}{\sqrt{3}}\frac{r_h H_0}{\sigma_v} . \tag{10}$$

This measure of the crossing time is equal to 0.343 times the collapse time of a growing homogeneous spherical perturbation. Groups with collapse times less than the Hubble time will have crossing times less than $0.343 H_0^{-1}$. Groups with longer crossing times cannot be expected to be virialized.

Virial mass is estimated as

$$M_{\rm VT} = \frac{6\sigma_v^2 r_h}{G} . \tag{11}$$

Table 1: Internal Properties of Raw HG Vm Groups*

	Mean	1st Quartile	Median	3rd Quartile
N	3.80 ± 0.22	3.00 ± 0.00	3.00 ± 0.00	4.10 ± 0.30
$\log_{10}(M_{\rm true}/M_\odot)$	13.54 ± 0.25	13.40 ± 0.15	13.82 ± 0.17	14.20 ± 0.14
$\log_{10}(M_{\rm VT}/M_\odot)$	13.91 ± 0.12	13.45 ± 0.14	13.98 ± 0.14	14.41 ± 0.17
$\log_{10}(M_{\rm VT}/M_{\rm true})$	0.366 ± 0.262	-0.224 ± 0.067	0.159 ± 0.121	0.620 ± 0.122
r_h (Mpc)	1.23 ± 0.14	0.81 ± 0.12	1.14 ± 0.15	1.57 ± 0.20
σ_v (km s^{-1})	291.3 ± 25.5	154.8 ± 28.5	259.7 ± 22.3	391.5 ± 47.0
$H_0\sigma_r$ (km s^{-1})	242.2 ± 57.1	51.7 ± 19.8	155.7 ± 61.7	370.4 ± 108.9
$H_0 t_c$	0.431 ± 0.119	0.159 ± 0.020	0.247 ± 0.017	0.435 ± 0.057
$M_{\rm true}/L$ $(M_\odot/L_\odot)$	346.0 ± 202.7	142.2 ± 10.2	207.4 ± 18.9	328.7 ± 56.7
$M_{\rm VT}/L$ $(M_\odot/L_\odot)$	1041.6 ± 1450.8	135.9 ± 34.6	369.0 ± 84.2	773.3 ± 211.9

* Dependences on h have been included in the quoted values for this and subsequent tables. We use $h = 0.5$.

Table 2: Internal Properties of Breakup HG Vm Groups

	Mean	1st Quartile	Median	3rd Quartile
N	6.43 ± 1.01	3.00 ± 0.00	4.50 ± 0.67	7.20 ± 1.29
$\log_{10}(M_{\rm true}/M_\odot)$	13.98 ± 0.11	13.70 ± 0.10	14.02 ± 0.11	14.37 ± 0.08
$\log_{10}(M_{\rm VT}/M_\odot)$	13.99 ± 0.15	13.52 ± 0.16	14.05 ± 0.18	14.53 ± 0.17
$\log_{10}(M_{\rm VT}/M_{\rm true})$	0.011 ± 0.120	-0.360 ± 0.084	0.003 ± 0.075	0.318 ± 0.098
r_h (Mpc)	0.95 ± 0.12	0.53 ± 0.08	0.80 ± 0.08	1.24 ± 0.23
σ_v (km s^{-1})	368.1 ± 39.4	204.7 ± 33.8	329.4 ± 29.3	497.4 ± 57.6
$H_0\sigma_r$ (km s^{-1})	164.4 ± 45.6	14.9 ± 2.8	42.5 ± 14.3	225.8 ± 103.22
$H_0 t_c$	0.217 ± 0.039	0.090 ± 0.007	0.143 ± 0.020	0.246 ± 0.031
$M_{\rm true}/L$ $(M_\odot/L_\odot)$	456.9 ± 87.9	251.1 ± 35.2	347.9 ± 27.0	454.6 ± 47.8
$M_{\rm VT}/L$ $(M_\odot/L_\odot)$	525.6 ± 115.2	153.5 ± 27.9	334.0 ± 43.1	666.3 ± 118.5

Table 3: Internal Properties of Raw NW Vm Groups

	Mean	1st Quartile	Median	3rd Quartile
N	3.42 ± 0.12	3.00 ± 0.00	3.00 ± 0.00	3.90 ± 0.30
$log_{10}(M_{\rm true}/M_\odot)$	13.65 ± 0.27	13.35 ± 0.25	13.82 ± 0.22	14.19 ± 0.19
$\log_{10}(M_{\rm VT}/M_\odot)$	13.47 ± 0.15	13.09 ± 0.18	13.53 ± 0.17	13.93 ± 0.16
$\log_{10}(M_{\rm VT}/M_{\rm true})$	-0.183 ± 0.230	-0.651 ± 0.132	-0.240 ± 0.085	0.095 ± 0.084
r_h (Mpc)	1.18 ± 0.15	0.71 ± 0.10	1.06 ± 0.14	1.45 ± 0.20
σ_v (km s^{-1})	170.9 ± 18.8	99.9 ± 12.8	160.9 ± 19.7	234.1 ± 31.0
$H_0\sigma_r$ (km s^{-1})	150.8 ± 46.0	35.8 ± 10.0	89.2 ± 25.8	253.8 ± 88.9
$H_0 t_c$	0.560 ± 0.067	0.245 ± 0.033	0.391 ± 0.036	0.682 ± 0.105
$M_{\rm true}/L$ $(M_\odot/L_\odot)$	303.7 ± 77.7	142.0 ± 15.3	211.1 ± 29.2	336.9 ± 55.8
$M_{\rm VT}/L$ $(M_\odot/L_\odot)$	1468.3 ± 2571.7	47.9 ± 17.2	112.0 ± 24.6	278.3 ± 81.4

Table 4: Internal Properties of Breakup NW Vm Groups

	Mean	1st Quartile	Median	3rd Quartile
N	5.14 ± 0.55	3.00 ± 0.00	3.80 ± 0.40	5.53 ± 0.63
$\log_{10}(M_{\rm true}/M_\odot)$	13.98 ± 0.16	13.74 ± 0.11	14.05 ± 0.11	14.39 ± 0.10
$\log_{10}(M_{\rm VT}/M_\odot)$	13.40 ± 0.09	13.04 ± 0.09	13.42 ± 0.06	13.82 ± 0.12
$\log_{10}(M_{\rm VT}/M_{\rm true})$	-0.575 ± 0.122	-0.965 ± 0.098	-0.587 ± 0.038	-0.257 ± 0.065
r_h (Mpc)	0.87 ± 0.10	0.45 ± 0.05	0.68 ± 0.07	1.04 ± 0.11
σ_v (km s^{-1})	187.3 ± 13.8	118.5 ± 15.1	171.1 ± 10.4	238.1 ± 16.8
$H_0\sigma_r$ (km s^{-1})	81.9 ± 15.2	11.4 ± 2.0	21.8 ± 2.5	66.6 ± 19.3
$H_0 t_c$	0.361 ± 0.081	0.143 ± 0.020	0.230 ± 0.030	0.423 ± 0.077
$M_{\rm true}/L$ $(M_\odot/L_\odot)$	786.6 ± 211.0	364.3 ± 40.2	518.4 ± 59.3	827.0 ± 164.6
$M_{\rm VT}/L$ $(M_\odot/L_\odot)$	147.1 ± 81.3	57.3 ± 15.1	119.8 ± 25.4	249.6 ± 43.1

Table 5: Internal Properties of Raw HG Rm Groups

	Mean	1st Quartile	Median	3rd Quartile
N	3.40 ± 0.23	3.00 ± 0.00	3.00 ± 0.00	3.55 ± 0.43
$log_{10}(M_{\rm true}/M_{\odot})$	13.93 ± 0.21	13.53 ± 0.29	14.00 ± 0.24	14.34 ± 0.22
$log_{10}(M_{\rm VT}/M_{\odot})$	13.92 ± 0.14	13.43 ± 0.20	14.00 ± 0.17	14.51 ± 0.13
$log_{10}(M_{\rm VT}/M_{\rm true})$	-0.006 ± 0.189	-0.353 ± 0.200	0.036 ± 0.132	0.406 ± 0.145
r_h (Mpc)	1.16 ± 0.17	0.80 ± 0.13	1.10 ± 0.18	1.51 ± 0.28
σ_v (km s^{-1})	319.7 ± 39.0	151.8 ± 32.7	276.7 ± 33.7	437.9 ± 66.0
$H_0\sigma_r$ (km s^{-1})	25.3 ± 4.0	15.0 ± 4.9	24.7 ± 4.0	33.4 ± 5.3
$H_0 t_c$	0.401 ± 0.120	0.147 ± 0.044	0.226 ± 0.049	0.402 ± 0.081
$M_{\rm true}/L$ $(M_{\odot}/L_{\odot})$	347.2 ± 119.0	187.8 ± 55.8	267.2 ± 74.2	364.5 ± 87.6
$M_{\rm VT}/L$ $(M_{\odot}/L_{\odot})$	632.1 ± 463.9	113.3 ± 61.2	276.9 ± 132.6	857.7 ± 705.2

Table 6: Internal Properties of Breakup HG Rm Groups

	Mean	1st Quartile	Median	3rd Quartile
N	6.91 ± 1.36	3.30 ± 0.46	4.90 ± 0.94	7.68 ± 1.69
$log_{10}(M_{\rm true}/M_{\odot})$	14.02 ± 0.07	13.71 ± 0.08	13.99 ± 0.09	14.36 ± 0.09
$log_{10}(M_{\rm VT}/M_{\odot})$	13.95 ± 0.09	13.55 ± 0.10	14.00 ± 0.10	14.47 ± 0.10
$log_{10}(M_{\rm VT}/M_{\rm true})$	-0.066 ± 0.066	-0.324 ± 0.084	0.019 ± 0.074	0.263 ± 0.072
r_h (Mpc)	0.75 ± 0.11	0.43 ± 0.05	0.67 ± 0.07	0.99 ± 0.15
σ_v (km s^{-1})	385.0 ± 30.4	218.1 ± 25.1	339.5 ± 29.3	526.2 ± 45.7
$H_0\sigma_r$ (km s^{-1})	19.2 ± 1.9	10.7 ± 1.9	17.3 ± 1.7	25.3 ± 2.5
$H_0 t_c$	0.181 ± 0.058	0.066 ± 0.009	0.112 ± 0.021	0.193 ± 0.044
$M_{\rm true}/L$ $(M_{\odot}/L_{\odot})$	427.2 ± 80.4	289.3 ± 17.7	354.3 ± 10.9	432.2 ± 34.5
$M_{\rm VT}/L$ $(M_{\odot}/L_{\odot})$	490.4 ± 82.0	164.4 ± 59.7	335.1 ± 69.9	664.8 ± 141.8

Table 7: Internal Properties of Raw NW Rm Groups

	Mean	1st Quartile	Median	3rd Quartile
N	3.29 ± 0.23	3.00 ± 0.00	3.00 ± 0.00	3.10 ± 0.30
$log_{10}(M_{\rm true}/M_{\odot})$	14.02 ± 0.18	13.69 ± 0.35	14.08 ± 0.24	14.42 ± 0.13
$log_{10}(M_{\rm VT}/M_{\odot})$	14.01 ± 0.16	13.54 ± 0.25	14.09 ± 0.24	14.58 ± 0.16
$log_{10}(M_{\rm VT}/M_{\rm true})$	-0.011 ± 0.171	-0.309 ± 0.153	0.039 ± 0.141	0.346 ± 0.173
r_h (Mpc)	1.16 ± 0.15	0.75 ± 0.12	1.03 ± 0.18	1.45 ± 0.21
σ_v (km s^{-1})	353.3 ± 48.3	181.1 ± 63.5	305.3 ± 60.7	475.9 ± 83.8
$H_0\sigma_r$ (km s^{-1})	26.3 ± 3.8	14.3 ± 3.8	24.8 ± 3.7	35.1 ± 6.8
H_0t_c	0.359 ± 0.102	0.126 ± 0.039	0.198 ± 0.038	0.360 ± 0.107
$M_{\rm true}/L$ $(M_{\odot}/L_{\odot})$	276.5 ± 84.2	143.4 ± 24.0	215.8 ± 52.8	332.8 ± 61.0
$M_{\rm VT}/L$ $(M_{\odot}/L_{\odot})$	599.5 ± 482.3	104.4 ± 77.6	218.9 ± 127.2	843.2 ± 754.6

Table 8: Internal Properties of Breakup NW Rm Groups

	Mean	1st Quartile	Median	3rd Quartile
N	6.70 ± 1.14	3.12 ± 0.30	4.55 ± 0.85	7.60 ± 1.43
$log_{10}(M_{\rm true}/M_{\odot})$	13.99 ± 0.12	13.70 ± 0.10	14.00 ± 0.10	14.37 ± 0.10
$log_{10}(M_{\rm VT}/M_{\odot})$	13.93 ± 0.09	13.51 ± 0.13	13.97 ± 0.09	14.44 ± 0.13
$log_{10}(M_{\rm VT}/M_{\rm true})$	-0.064 ± 0.117	-0.376 ± 0.077	0.003 ± 0.069	0.237 ± 0.089
r_h (Mpc)	0.71 ± 0.11	0.42 ± 0.04	0.63 ± 0.07	0.89 ± 0.14
σ_v (km s^{-1})	387.2 ± 32.1	215.4 ± 23.7	341.7 ± 24.1	528.1 ± 46.1
$H_0\sigma_r$ (km s^{-1})	17.9 ± 1.8	10.2 ± 1.4	16.0 ± 1.5	23.5 ± 2.8
H_0t_c	0.179 ± 0.075	0.062 ± 0.010	0.105 ± 0.017	0.183 ± 0.049
$M_{\rm true}/L$ $(M_{\odot}/L_{\odot})$	460.1 ± 99.3	308.9 ± 17.0	366.5 ± 21.4	461.1 ± 37.8
$M_{\rm VT}/L$ $(M_{\odot}/L_{\odot})$	435.3 ± 172.5	164.2 ± 64.0	334.1 ± 70.1	658.2 ± 140.9

Table 9: Internal Properties of Raw RM Groups

	Mean	1st Quartile	Median	3rd Quartile
N	3.90 ± 0.05	3.00 ± 0.00	3.00 ± 0.00	4.00 ± 0.00
$log_{10}(M_{\rm true}/M_\odot)$	12.70 ± 0.02	12.25 ± 0.02	12.60 ± 0.02	13.11 ± 0.03
$log_{10}(M_{\rm VT}/M_\odot)$	12.95 ± 0.05	12.38 ± 0.05	12.99 ± 0.04	13.56 ± 0.06
$log_{10}(M_{\rm VT}/M_{\rm true})$	0.245 ± 0.038	-0.225 ± 0.022	0.209 ± 0.024	0.686 ± 0.047
r_h (Mpc)	0.98 ± 0.03	0.72 ± 0.02	0.97 ± 0.03	1.23 ± 0.04
σ_v (km s^{-1})	125.2 ± 5.9	45.0 ± 1.8	89.3 ± 3.7	162.4 ± 8.8
$H_0\sigma_r$ (km s^{-1})	19.2 ± 0.1	12.7 ± 0.1	18.3 ± 0.2	24.8 ± 0.3
$H_0 t_c$	1.095 ± 0.099	0.317 ± 0.011	0.590 ± 0.024	1.144 ± 0.026
$M_{\rm true}/L$ $(M_\odot/L_\odot)$	289.6 ± 2.5	193.0 ± 1.7	229.1 ± 2.9	315.0 ± 6.1
$M_{\rm VT}/L$ $(M_\odot/L_\odot)$	2001.6 ± 265.4	154.4 ± 8.5	456.8 ± 20.8	1337.6 ± 109.5

Table 10: Internal Properties of Breakup RM Groups

	Mean	1st Quartile	Median	3rd Quartile
N	24.69 ± 1.10	3.00 ± 0.00	14.00 ± 0.89	29.95 ± 2.10
$log_{10}(M_{\rm true}/M_\odot)$	13.35 ± 0.02	13.04 ± 0.06	13.36 ± 0.02	13.68 ± 0.02
$log_{10}(M_{\rm VT}/M_\odot)$	13.41 ± 0.03	13.12 ± 0.03	13.48 ± 0.03	13.79 ± 0.03
$log_{10}(M_{\rm VT}/M_{\rm true})$	0.052 ± 0.018	-0.142 ± 0.018	0.020 ± 0.015	0.193 ± 0.017
r_h (Mpc)	0.62 ± 0.02	0.39 ± 0.01	0.52 ± 0.02	0.78 ± 0.03
σ_v (km s^{-1})	232.9 ± 5.5	134.3 ± 7.9	222.1 ± 6.0	299.0 ± 6.2
$H_0\sigma_r$ (km s^{-1})	16.6 ± 0.4	11.3 ± 0.2	15.3 ± 0.5	20.2 ± 0.5
$H_0 t_c$	0.354 ± 0.040	0.082 ± 0.004	0.120 ± 0.003	0.325 ± 0.037
$M_{\rm true}/L$ $(M_\odot/L_\odot)$	492.0 ± 44.6	318.9 ± 2.3	363.5 ± 3.2	428.4 ± 5.5
$M_{\rm VT}/L$ $(M_\odot/L_\odot)$	1325.1 ± 218.0	252.6 ± 10.0	362.2 ± 9.1	622.8 ± 30.6

Table 11: Internal Properties of CfA HG Groups

	Mean	1st Quartile	Median	3rd Quartile
N	6.67	3.00	4.00	6.00
$\log_{10}(M_{\rm VT}/M_{\odot})$	13.70	12.93	13.71	14.47
r_h (Mpc)	1.08	0.38	0.81	1.67
σ_v (km s^{-1})	290.8	121.5	215.3	398.3
$H_0 t_c$	0.288	0.117	0.218	0.352
$M_{\rm VT}/L$ $(M_{\odot}/L_{\odot})$	3365	38	204	572

Table 12: Internal Properties of CfA NW Groups

	Mean	1st Quartile	Median	3rd Quartile
N	5.59	3.00	4.00	6.00
$\log_{10}(M_{\rm VT}/M_{\odot})$	13.22	12.74	13.27	13.85
r_h (Mpc)	0.85	0.35	0.64	1.13
σ_v (km s^{-1})	177.2	78.3	153.0	241.4
$H_0 t_c$	0.404	0.155	0.250	0.461
$M_{\rm VT}/L$ $(M_{\odot}/L_{\odot})$	973	20	55	188

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Fig. 1.— $M_{\rm halo}/L$ vs $M_{\rm halo}$ for the halos in the simulation.

Fig. 2.— Two point correlation functions for simulated halos and groups in the raw catalogs, each calculated for ten catalogs corresponding to different observers and averaged over those ten catalogs. Arrows on the s axis near 0.7 and 0.8 indicate the correlation lengths quoted by Ramella et al. (1990) for galaxies and for groups, respectively. A representative error bar for ξ_{GG} is also plotted. Error bars for ξ_{gg} are much smaller. The symbol key gives absolute magnitude limit for the RM catalogs and the value of the linking length used for the groups. D_0 is the transverse linking length for the Vm and Rm groups and the total three dimensional linking length for the RM groups. Correlation functions for NW groups, not shown here, are noisier but otherwise indistinguishable from those of HG groups.

Fig. 3.— Same as Fig. 2, but for the breakup catalogs.